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Finally, since for values of x greater than x_n , $n^2 x^{2n-2} > 2$, we may develop $\sqrt{1 + n^2 x^{2n-2}}$ into a series in descending powers of $n^2 x^{2n-2}$, and the series will be uniformly and absolutely convergent for values of x between, and including, x_n and 1, and all values of $n \geq N$, we have

$$\sqrt{1 + n^2 x^{2n-2}} = nx^{n-1} + \frac{1}{2} \cdot \frac{1}{nx^{n-1}} - \frac{1}{2 \cdot 4} \cdot \frac{1}{n^3 x^{3n-3}} + \dots$$

Hence,

$$(5) \quad \int_{x_n}^1 \sqrt{1 + n^2 x^{2n-2}} \cdot dx = 1 - x_n^n + \left[\frac{1}{2} \cdot \frac{1}{n(-n+2)} x^{-n+2} - + \dots \right]_{x_n}^1.$$

Since,

$$(6) \quad nx_n^{n-1} = \sqrt{2},$$

$$(7) \quad \lim_{n \rightarrow \infty} x_n^n = 0.$$

We may now obtain the limit of the right hand side of (5) when $n \rightarrow \infty$ by taking the limit of each term. This gives, in consequence of (6) and (7), 1 as the limit. In other words, we may choose n so large that

$$(8) \quad \int_{x_n}^1 \sqrt{1 + n^2 x^{2n-2}} \cdot dx = 1 + \zeta,$$

where ζ is a constant as small numerically as we please for all values of $n > N$.

Then

$$\begin{aligned} \left| \int_0^1 \sqrt{1 + n^2 x^{2n-2}} \cdot dx - 2 \right| &\leq \left| \int_0^{1-\epsilon} \sqrt{1 + n^2 x^{2n-2}} \cdot dx - 1 \right| + \left| \int_{1-\epsilon}^{x_n} \sqrt{1 + n^2 x^{2n-2}} \cdot dx \right| \\ &\quad + \left| \int_{x_n}^1 \sqrt{1 + n^2 x^{2n-2}} \cdot dx - 1 \right| < \frac{\eta}{2} + \epsilon + \sqrt{3} \cdot \epsilon + |\zeta|. \end{aligned}$$

Since this may be made as small as we please by a suitable choice in order of ϵ , η , and ζ , we have proved the statement.

Also solved by TOBIAS DANTZIG.

MECHANICS.

312. Proposed by C. N. SCHMALL, New York City.

A ball of elasticity e is projected upward from a point on an inclined plane, so that after its first contact with the plane it rebounds to its starting point. If ϕ be the inclination of the plane to the horizontal, and ψ the angle made by the line of projection with the inclined plane, show that

$$\cot \phi \cot \psi = e + 1.$$

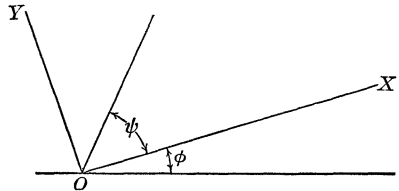
I. SOLUTION BY HORACE OLSON, Chicago, Illinois.

In order that the answer given may be true the ball must be thrown in a vertical plane perpendicular to the inclined plane. Taking the axis of coordinates as indicated in the figure, the equations of the trajectory are:

$$x = vt \cos \psi - \frac{gt^2 \sin \phi}{2},$$

$$y = vt \sin \psi - \frac{gt^2 \cos \phi}{2},$$

t being the time from the instant of projection. Then,



$$\frac{dx}{dt} = v \cos \psi - gt \sin \phi \quad \text{and} \quad \frac{dy}{dt} = v \sin \psi - gt \cos \phi.$$

When the ball strikes the plane, $y = 0$,

$$t = \frac{2v \sin \psi}{g \cos \phi}, \quad \text{and } x \text{ becomes } x_1 = \frac{2v^2 \sin \psi}{g \cos^2 \phi} (\cos \psi \cos \phi - \sin \psi \sin \phi),$$

whence,

$$\frac{dx_1}{dt} = v \cos \psi - \frac{2v \sin \psi \sin \phi}{\cos \phi}, \quad \frac{dy_1}{dt} = -v \sin \psi.$$

After the rebound,

$$x = x_1 + vt \cos \psi - \frac{2vt \sin \psi \sin \phi}{\cos \phi} - \frac{gt^2 \sin \phi}{2}, \quad y = evt \sin \psi - \frac{gt^2 \cos \phi}{2},$$

t being here reckoned from the instant of rebound. When the ball again strikes the plane, $y = 0$, and

$$t = \frac{2ev \sin \psi}{g \cos \phi} \text{ and } x \text{ becomes } x_2 = x_1 + \frac{2ev^2 \sin \psi \cos \psi}{g \cos \phi} - \frac{4ev^2 \sin^2 \psi \sin \phi}{g \cos^2 \phi} - \frac{2e^2 v^2 \sin^2 \psi \sin \phi}{g \cos^2 \phi},$$

or

$$x_2 = \frac{2v^2 \sin \psi}{g \cos^2 \phi} \{ (e+1) \cos \psi \cos \phi - (e^2 + 2e + 1) \sin \psi \sin \phi \}.$$

By the conditions of the problem, $x_2 = 0$, and, hence, $\cot \phi \cot \psi = e + 1$.

II. SOLUTION BY THE PROPOSER.

Let v be the velocity of projection; then $v \sin \psi$ and $v \cos \psi$ are the components respectively perpendicular and parallel to the plane; and the components of gravity perpendicular and parallel to the plane are $g \cos \phi$ and $g \sin \phi$. Considering the perpendicular component, the time of flight is twice the time in which the velocity $v \sin \psi$ would be acquired under the action of the force $g \cos \phi$, *i. e.*, $2v \sin \psi / g \cos \phi$. After rebounding, the ball's velocity perpendicular to the plane is $ev \sin \psi$; hence, the time required in returning to the point of projection is $2ev \sin \psi / g \cos \phi$; and the whole time of flight is

$$2 \frac{v \sin \psi}{g \cos \phi} (e + 1).$$

Now, since the motion parallel to the plane is not affected by the impact, the entire time of flight is equal to twice the time in which the velocity $v \cos \psi$ would be produced by the force $g \sin \phi$, *i. e.*, $2 \frac{v \cos \psi}{g \sin \phi}$. Therefore, equating the two expressions found for the time, we have

$$2 \frac{v \cos \psi}{g \sin \phi} = 2 \frac{v \sin \psi}{g \cos \phi} (e + 1);$$

whence,

$$\cot \phi \cot \psi = e + 1.$$

NUMBER THEORY.

An excellent solution of 226 by Mrs. ELIZABETH BROWN DAVIS should have been reported in the April issue. EDITORS.

227. Proposed by R. P. BAKER, University of Iowa.

Show that every rational number can be expressed as a finite sum $\sum_{n=m}^{n=m+k} \frac{a_n}{n}$; where a_n is either 0 or 1 and m is any positive integer.

SOLUTION BY FRANK IRWIN, University of California.

Since the series $\sum_{n=m}^{\infty} \frac{1}{n}$ is divergent, we can find a value of r such that our given rational number, N , lies between $1/m + 1/(m+1) + \dots + 1/(r-1)$ and $1/m + 1/(m+1) + \dots + 1/r$ (unless indeed N is equal to an expression of this sort, in which case our problem is solved). This reduces the problem to that of expressing $N' = N - [1/m + 1/(m+1) + \dots + 1/(r-1)]$, that is, a (proper) fraction $< 1/r$, as $\sum_{n=r}^{n=m+k} \frac{a_n}{n}$.